

ON GAUGE FIXING IN THE LAGRANGIAN FORMALISM OF SUPERFIELD BRST QUANTIZATION

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We propose a modification of the gauge-fixing procedure in the Lagrangian method of superfield BRST quantization for general gauge theories which simultaneously provides a natural generalization of the well-known BV quantization scheme as far as gauge-fixing is concerned. A superfield form of BRST symmetry for the vacuum functional is found. The gauge-independence of the S-matrix is established.

1. Introduction

Recently, there has been a fairly large amount of papers [1, 2, 3, 4, 5] devoted to various superfield extensions of the BV quantization method [7] for gauge theories. Thus, in [1] the geometrical representation of BRST transformations [8] in form of supertranslations in superspace was realized, in [2, 3] the superspace formulation of Yang-Mills theories actions and BRST transformations was found, in [4] the superfield representation of generating operator Δ of the BV method was suggested, in [5] a closed superfield form of the BV quantization method [7] was found. In the study of [6] a multilevel generalization of the BV quantization formalism was proposed, which ensures an invariant description of the formalism in terms of field-antifields variables.

It is well-known that performing the quantization of a gauge theory is necessarily related to introducing a gauge-fixing procedure needed for constructing the Green functions and the S-matrix. The methods [5, 7] implement gauge-fixing underlied by an appropriate choice of the *fermionic* functional, with the only restriction being the non-degeneracy of the corresponding generating functional.

In the framework of the multilevel generalization [6] of the BV formalism a gauge-fixing procedure has been introduced by means of hypergauge functions depending on the entire set of field-antifield variables. The hypergauge functions enter the quantum action in *linear* combinations with the corresponding Lagrange multipliers and are subject to certain involution relations expressed in terms of the same antibracket operation that occurs in the generating equation determining the quantum action. These involution relations serve to ensure the BRST invariance of the vacuum functional, thus providing effectively for its independence from hypergauge variations of canonical form in the antibracket sense.

In this paper we consider another generalization of the BV quantization scheme obtained by modifying the superfield formalism [5] in such a way that the gauge is now introduced with the help of a gauge-fixing *bosonic* functional which depends generally on the entire set of variables of the formalism, including sources and Lagrange multipliers, and which is subjected to a generating equation formally analogous to the equation determining the quantum action. On the one hand, this approach to gauge-fixing guarantees

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the independence of the vacuum functional (and hence of the S-matrix) from any particular choice of the gauge. On the other hand, within this framework, as compared to that of [6], we are no longer confined to the linear dependence on the Lagrange multiplier fields. Remarkably, the generating equation introducing the gauge admits of a solution which is identical with the gauge-fixing functional found in the original version of [5], and which, in particular, includes the well-known gauge-fixing condition of the BV formalism.

Our starting point at the classical level is a general gauge theory with the well-known structure of the configuration space $\phi^A, \varepsilon(\phi^A) = \varepsilon_A$, described by the rules [7], depending on whether the theory is a reducible or irreducible one.

We use the condensed notations [9] and the conventions adopted in [5].

2. Modified Superfield BRST Quantization

In this section we shall extend the procedure [5] of superfield quantization underlied by the principle of BRST invariance. We first introduce a superspace (x^μ, θ) spanned by space-time coordinates x^μ , $\mu = (0, 1, \dots, D-1)$, and a scalar anticommuting coordinate θ . Let $\Phi^A(\theta)$ be a set of superfields, which is accompanied by a set of the corresponding super-antifields $\Phi_A^*(\theta)$ with the Grassmann parities $\varepsilon(\Phi^A) \equiv \varepsilon_A$, $\varepsilon(\Phi_A^*) = \varepsilon_A + 1$, and is subjected to the boundary condition

$$\Phi^A(\theta)|_{\theta=0} = \phi^A. \quad (1)$$

We define the vacuum functional Z as the following functional integral:

$$Z = \int d\Phi d\Phi^* \rho[\Phi^*] \exp \left\{ \frac{i}{\hbar} \left(W[\Phi, \Phi^*] + X[\Phi, \Phi^*] + \Phi^* \Phi \right) \right\}. \quad (2)$$

Here, $W = W[\Phi, \Phi^*]$ is the quantum action which obeys the generating equation

$$\frac{1}{2}(W, W) + VW = i\hbar \Delta W, \quad (3)$$

while the (bosonic) gauge-fixing functional $X = X[\Phi, \Phi^*]$ is required to satisfy the equation

$$\frac{1}{2}(X, X) - UX = i\hbar \Delta X. \quad (4)$$

In Eqs. (2), (3), (4), we have used the antibracket $(\ , \)$ expressed in terms of arbitrary functionals $F = F[\Phi, \Phi^*]$, $G = G[\Phi, \Phi^*]$ by the rule [5]

$$(F, G) = \int d\theta \left\{ \frac{\delta F}{\delta \Phi^A(\theta)} \frac{\partial}{\partial \theta} \frac{\delta G}{\delta \Phi_A^*(\theta)} (-1)^{\varepsilon_A+1} - (-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)} (F \leftrightarrow G) \right\}. \quad (5)$$

We also have used the operators Δ , U and V

$$\Delta = - \int d\theta (-1)^{\varepsilon_A} \frac{\delta_l}{\delta \Phi^A(\theta)} \frac{\partial}{\partial \theta} \frac{\delta}{\delta \Phi_A^*(\theta)}, \quad (6)$$

$$V = - \int d\theta \frac{\partial \Phi_A^*(\theta)}{\partial \theta} \frac{\delta}{\delta \Phi_A^*(\theta)}, \quad (7)$$

$$U = - \int d\theta \frac{\partial \Phi^A(\theta)}{\partial \theta} \frac{\delta_l}{\delta \Phi^A(\theta)} \quad (8)$$

(derivatives with respect to θ are understood as acting from the left) as well as the functionals $\rho[\Phi^*]$ and $\Phi^*\Phi$,

$$\rho[\Phi^*] = \delta\left(\int d\theta \Phi^*(\theta)\right), \quad (9)$$

$$\Phi^*\Phi = \int d\theta \Phi_A^*(\theta) \Phi^A(\theta). \quad (10)$$

The algebraic properties of both the antibracket (5) and the operators (6), (7), (8) were studied in detail in [5]. The algebra of the above operators has the form

$$\Delta^2 = 0, \quad U^2 = 0, \quad V^2 = 0, \quad (11)$$

$$UV + VU = 0, \quad \Delta U + U\Delta = 0, \quad \Delta V + V\Delta = 0. \quad (12)$$

It is convenient to recast the equations (3), (4) into the equivalent form

$$\bar{\Delta} \exp\left\{\frac{i}{\hbar} W\right\} = 0, \quad (13)$$

$$\tilde{\Delta} \exp\left\{\frac{i}{\hbar} X\right\} = 0, \quad (14)$$

using the operators

$$\bar{\Delta} = \Delta + \frac{i}{\hbar} V, \quad \tilde{\Delta} = \Delta - \frac{i}{\hbar} U, \quad (15)$$

whose algebra, by virtue of the properties (11), (12), reads as follows:

$$\bar{\Delta}^2 = 0, \quad \tilde{\Delta}^2 = 0, \quad \bar{\Delta}\tilde{\Delta} + \tilde{\Delta}\bar{\Delta} = 0. \quad (16)$$

Using the nilpotency of the operator U , we observe that any functional $X = U\Psi[\Phi]$, with $\Psi[\Phi]$ being an arbitrary fermionic functional, is obviously a solution of Eq. (4). The above expression gives the precise form of the gauge-fixing functional proposed by the study of [5] when formulating superfield BRST quantization.

A remarkable property of the integrand in (2) is its invariance under the following transformations of global supersymmetry with an anticommuting parameter μ :

$$\begin{aligned} \delta\Phi^A(\theta) &= \mu U\Phi^A(\theta) + (\Phi^A(\theta), X - W)\mu, \\ \delta\Phi_A^*(\theta) &= \mu V\Phi_A^*(\theta) + (\Phi_A^*(\theta), X - W)\mu. \end{aligned} \quad (17)$$

Here, we have taken into account, first, the expressions of the derivatives

$$\frac{\delta_l \Phi^A(\theta)}{\delta \Phi^B(\theta')} = (-1)^{\epsilon_A} \delta(\theta' - \theta) \delta_B^A = (-1)^{\epsilon_A} \frac{\delta \Phi^A(\theta)}{\delta \Phi^B(\theta')},$$

$$\frac{\delta \Phi_A^*(\theta)}{\delta \Phi_B^*(\theta')} = (-1)^{\epsilon_A+1} \delta(\theta' - \theta) \delta_A^B,$$

following from the definition of integration over the Grassmann variable θ

$$\int d\theta \theta = 1, \quad \int d\theta = 0, \quad F(\theta) = \int d\theta' \delta(\theta' - \theta) F(\theta'),$$

$$\delta(\theta' - \theta) = \theta' - \theta,$$

second, the invariance of the weight functional (9) under the transformations (17), $\delta\rho[\Phi^*] = 0$, third, the fact that under these transformations we have

$$\delta(W + X + \Phi^*\Phi) = 2\mu\left(\frac{1}{2}(W, W) + VW - \frac{1}{2}(X, X) + UX\right), \quad (18)$$

and, finally, the fact that the corresponding Jacobian Y has the form

$$Y = \exp(2\mu\Delta W - 2\mu\Delta X). \quad (19)$$

Eqs. (17) are the transformations of BRST symmetry in the framework of superfield quantization based on gauge-fixing X introduced as a solution of the corresponding generating equation (4).

We now consider the gauge-dependence of the vacuum functional Z , Eq. (2). Note, in the first place, that any admissible variation δX of the gauge-fixing functional X should satisfy the equation

$$(X, \delta X) - U\delta X = i\hbar\Delta\delta X, \quad (20)$$

which can be represented in the form

$$\hat{Q}(X)\delta X = 0. \quad (21)$$

Here, we have introduced the graded linear, nilpotent operator $\hat{Q}(X)$,

$$\hat{Q}(X) = \hat{\mathcal{B}}(X) - i\hbar\tilde{\Delta}, \quad \hat{Q}^2(X) = 0, \quad (22)$$

where $\hat{\mathcal{B}}(X)$ stands for an operator acting by the rule

$$(X, F) \equiv \hat{\mathcal{B}}(X)F, \quad (23)$$

and possessing the property

$$\hat{\mathcal{B}}^2(X) = \hat{\mathcal{B}}\left(\frac{1}{2}(X, X)\right). \quad (24)$$

By the nilpotency of the operator $\hat{Q}(X)$, any functional of the form

$$\delta X = \hat{Q}(X)\delta\Psi, \quad (25)$$

with $\delta\Psi$ being an arbitrary Fermionic functional, obeys the equation (21). Furthermore, as in the theorems proved by the study of Ref. [10], one can establish the fact that any solution δX of Eq. (21), vanishing when all the variables entering δX are equal to zero, is of the form (25) with a certain fermionic functional $\delta\Psi$.

Let $Z_X \equiv Z$ be the value of the vacuum functional (2) related to the gauge condition chosen as a functional X . In the vacuum functional $Z_{X+\delta X}$ we now make the change of variables (17) with some functional $\mu = \mu[\Phi, \Phi^*]$, accompanied by an additional change

$$\delta\Phi^A = (\Phi^A, \delta Y), \quad \delta\Phi_A^* = (\Phi_A^*, \delta Y), \quad \varepsilon(\delta Y) = 1, \quad (26)$$

where $\delta Y = -i\hbar\mu[\Phi, \Phi^*]$. We obtain

$$Z_{X+\delta X} = \int d\Phi d\Phi^* \rho[\Phi^*] \exp\left\{\frac{i}{\hbar}\left(W + X + \delta X + \delta X_1 + \Phi^*\Phi\right)\right\}, \quad (27)$$

In (27), we have denoted

$$\delta X_1 = 2 \left((X, \delta Y) - U \delta Y - i \hbar \Delta \delta Y \right) = 2 \hat{Q}(X) \delta Y. \quad (28)$$

Let the functional δY be chosen in the form (recall that $\delta X = \hat{Q}(X) \delta \Psi$)

$$\delta Y = -\frac{1}{2} \delta \Psi. \quad (29)$$

Thereby we find

$$Z_{X+\delta X} = Z_X, \quad (30)$$

which implies the fact that the vacuum functional (and, hence, the S-matrix, by the equivalence theorem [11]) does not depend on the gauge.

3. Discussion

In the previous section we have extended the superfield quantization method [5] for general gauge theories to encompass the concept of gauge-fixing introduced by means of an appropriate generating equation (see (4)).

The method of modified superfield BRST quantization permits one to generalize the BV quantization scheme [7] as far as the procedure of gauge-fixing is concerned. In fact, consider the component representation of the superfields $\Phi^A(\theta)$ and super-antifields $\Phi_A^*(\theta)$

$$\begin{aligned} \Phi^A(\theta) &= \phi^A + \lambda^A \theta, & \Phi_A^*(\theta) &= \phi_A^* - \theta J_A, \\ \varepsilon(\phi^A) &= \varepsilon(J_A) = \varepsilon_A, & \varepsilon(\phi_A^*) &= \varepsilon(\lambda^A) = \varepsilon_A + 1. \end{aligned}$$

The set of variables $\phi^A, \phi_A^*, \lambda^A, J_A$ is identical with the complete set of variables of the BV method [7].

Expressed in their component form, the antibracket (5) and the operator Δ (6) coincide with the corresponding objects of the BV method [7]

$$(F, G) = \frac{\delta F}{\delta \phi^A} \frac{\delta G}{\delta \phi_A^*} - (-1)^{(\varepsilon(F)+1)(\varepsilon(G)+1)} (F \leftrightarrow G),$$

$$\Delta = (-1)^{\varepsilon_A} \frac{\delta_l}{\delta \phi^A} \frac{\delta}{\delta \phi_A^*}.$$

Representing the integration measure in terms of the components

$$d\Phi d\Phi^* \rho[\Phi^*] = d\phi d\phi^* d\lambda dJ \delta(J),$$

we observe that solutions of the generating equations determining the action W when $J_A = 0$ may be sought among solutions of the master equation applied by the BV method

$$\frac{1}{2}(W, W) = i \hbar \Delta W, \quad (31)$$

since the operator V (7)

$$V = -J_A \frac{\delta}{\delta \phi_A^*}$$

vanishes when $J_A = 0$. Restricting ourselves to functionals W being independent of λ^A , and taking into account the component form of $\Phi^*\Phi$,

$$\Phi^*\Phi = \phi_A^*\lambda^A - J_A\phi^A,$$

we arrive at the following representation of the vacuum functional, Eq. (2):

$$Z = \int d\phi d\phi^* d\lambda \exp \left\{ \frac{i}{\hbar} \left[W(\phi, \phi^*) + X(\phi, \phi^*, \lambda) + \phi_A^*\lambda^A \right] \right\}. \quad (32)$$

The above result may be considered as an extension of the BV quantization procedure [7] to a more general case of gauge-fixing. In fact, as stated above, the functional $X = U\Psi[\Phi]$ is a solution of the generating equation (4). From the component representation of the operator U

$$U = -(-1)^{\varepsilon_A} \lambda^A \frac{\delta_l}{\delta \phi^A},$$

choosing the functional Ψ to be independent of the fields λ^A , $\Psi = \Psi(\phi)$, it follows that the gauge-fixing functional X

$$X(\phi, \lambda) = -\frac{\delta \Psi(\phi)}{\delta \phi^A} \lambda^A$$

becomes identical with the gauge applied by the BV quantization method.

It is instructive to compare the general framework of the present study with that of the multilevel formalism [6]. Notice that in writing down the relations of [6] we apply the notations of the present paper, thereby assuming an explicit separation of field-antifield variables in terms of the Darboux coordinates.

Assuming the measure density of the functional integration to be trivial, the general ansatz for the first-level vacuum functional [6] reads as follows:

$$Z = \int d\phi d\phi^* d\lambda \exp \left\{ \frac{i}{\hbar} \left[W(\phi, \phi^*) + G_A(\phi, \phi^*) \lambda^A \right] \right\}, \quad (33)$$

where $W(\phi, \phi^*)$ is a quantum action subject to the generating equations (31), while $G_A(\phi, \phi^*)$ are gauge-fixing functions, or hypergauge functions according to the terminology of [6].

The hypergauge functions $G_A = G_A(\phi, \phi^*)$ are assumed [6] to satisfy the involution relations

$$(G_A, G_B) = G_C U_{AB}^C \quad (34)$$

accompanied by the so-called unimodularity conditions

$$\Delta G_A - U_{BA}^B (-1)^{\varepsilon_B} = G_B V_A^B, \quad V_A^A = G_A \tilde{G}^A \quad (35)$$

with certain structure functions U_{AB}^C , V_A^B , \tilde{G}^A .

As shown in [6], the set of conditions (34), (35), combined with the generating equation (31), serves to ensure the invariance of the vacuum functional (33) under the BRST transformations

$$\begin{aligned} \delta \phi^A &= (\phi^A, W - G_B \lambda^B) \mu, & \delta \phi_A^* &= (\phi_A^*, W - G_B \lambda^B) \mu, \\ \delta \lambda^A &= \left(U_{BC}^A \lambda^C \lambda^B (-1)^{\varepsilon_B} - 2i\hbar V_B^A \lambda^B - 2(i\hbar)^2 \tilde{G}^A \right) \mu, \end{aligned} \quad (36)$$

which, in their turn, provide for the independence of the vacuum functional from hypergauge variations of canonical form

$$\delta G_A = (G_A, \delta Y), \quad \varepsilon(\delta Y) = 1. \quad (37)$$

It should be noted that the above variation (37) is compatible with the conditions (34), (35). Within the assumption that the hypergauge functions are solvable with respect to the antifields, the first-level vacuum functional (33) reduces to the well-known form [7] of the BV formalism.

In view of the above, we now return to the vacuum functional (32) of the modified superfield formalism and consider the particular case of linear dependence of the gauge-fixing functional $X(\phi, \phi^*, \lambda)$ on the Lagrange multipliers, i.e.

$$X(\phi, \phi^*, \lambda) + \phi_A^* \lambda^A \equiv G_A(\phi, \phi^*) \lambda^A.$$

Then, firstly, equation (4) determining X reduces to the conditions

$$(G_A, G_B) = 0, \quad \Delta G_A = 0, \quad (38)$$

and, secondly, the transformations of BRST symmetry for the vacuum functional (32), take on the form

$$\delta \phi^A = (\phi^A, G_B \lambda^B - W) \mu, \quad \delta \phi_A^* = (\phi_A^*, G_B \lambda^B - W) \mu, \quad \delta \lambda^A = 0, \quad (39)$$

which follows from (17) when $J_A = 0$.

Obviously, equations (38), are identical with (34), (35) in the particular case of vanishing structure functions U_{AB}^C , V_A^B , \tilde{G}^A , whereas the transformations (39), similarly, present a particular case of (36).

Summarizing, both the study of the present paper and that of [6] apply gauge-fixing conditions depending on the whole set of field-antifield variables; they both contain the BV method as a particular case and ensure the gauge-independence of the S-matrix. At the same time, in the present approach the gauge-independence is encoded in BRST transformations controlled by generating equations imposed on the entire gauge part of the quantum action, whereas in the framework of [6] this role is played by unimodular involution relations imposed on hypergauge functions. Despite the formal similarity between the vacuum functional (32) proposed by the present study and the ansatz (33) suggested by [6] the two methods appear to be independent from each another, being different generalizations of the BV method. However, as is seen from the above comparison, the vacuum functional (32) proposed by the present study becomes identical, in the particular case of linear dependence on the Lagrange multipliers, with the first-level vacuum functional [6] considered in the case of a trivial integration measure and vanishing structure functions. On the other hand, the vacuum functional (32) can be regarded as a formal extension of (33) in the sense that it admits of more than linear dependence on the Lagrange multipliers.

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References

- [1] L. Bonora and M. Tonin, *Phys. Lett.* **B98**, 48 (1981);
L. Bonora, P. Pasti and M. Tonin, *J. Math. Phys.* **23**, 839 (1982);
L. Baulieu, *Phys. Rep.* **129**, 1 (1985).
- [2] C.M. Hull, B. Spence and J.L. Vazquez-Bello, *Nucl. Phys.* **B348**, 108 (1991).
- [3] S.D. Joglekar and B.P. Mandal, *A superspace formulation of an "asymptotic" $OSp(3,1|2)$ invariance of Yang-Mills theories*, hep-th/9904106.
- [4] N.R.F. Braga and A. Das, *Nucl. Phys.* **B442**, 655 (1995);
N.R.F. Braga and S.M. de Souza, *Phys. Rev.* **D53**, 916 (1996);
E.M.C. Abreu and N.R.F. Braga, *Phys. Rev.* **D54**, 4080 (1996); *Int. J. Mod. Phys.* **A13**, 493 (1998).
- [5] P.M. Lavrov, P.Yu. Moshin and A.A. Reshetnyak, *Mod. Phys. Lett.* **A10**, 2687 (1995);
JETP Lett. **62**, 780 (1995).
- [6] I.A. Batalin and I.V. Tyutin, *Int. J. Mod. Phys.* **A8**, 2333 (1993).
- [7] I.A. Batalin and G.A. Vilkovisky, *Phys. Lett.* **B102**, 27 (1981); *Phys. Rev.* **D28** 2567 (1983).
- [8] C. Becchi, A. Rouet, R. Stora, *Phys. Lett.* **B25**, 344 (1974); *Comm. Math. Phys.* **42**, 127 (1975);
I.V. Tyutin, Lebedev Inst. preprint, No. 35 (1975).
- [9] B.S. de Witt, *Dynamical Theory of Groups and Fields* (Gordon and Breach, New York, 1965).
- [10] I.A. Batalin, P.M. Lavrov and I.V. Tyutin, *J. Math. Phys.* **31**, 1487 (1990); **32**, 2513 (1991).
- [11] R.E. Kallosh and I.V. Tyutin, *Yad. Fiz.* **17**, 190 (1973).